Sino–French Conference in Algebraic and Complex Geometry

April 23-27, 2018, Lyon, France

Titles and Abstracts

Mini-courses:

Bruno Klingler: *o-minimality and Hodge theory*

The idea of tame topology was introduced by Grothendieck in "Esquisse d'un programme" and developed by model theorists under the name "o-minimal structures". In these lectures I will define o-minimal structures, study their basic properties and their relevance to complex geometry, in particular Hodge theory. As an illustration I will explain that period maps are tame. As an immediate corollary one recovers the algebraicity of Hodge loci (a classical result of Cattani-Deligne-Kaplan).

Tony Yue Yu: *Non-archimedean SYZ fibration and enumerative geometry*

The motivations come from mirror symmetry and the SYZ conjecture concerning degenerations of Calabi-Yau manifolds. In the first talk, I will give an introduction to the SYZ fibration, and explain its analog in non-archimedean geometry, based on the works of Berkovich, Kontsevich-Soibelman, Nicaise-Mustata, Nicaise-Xu, and my joint work with Nicaise and Xu. In the second talk, I will explain enumeration of curves with boundaries in non-archimedean SYZ fibrations in the case of log Calabi-Yau surfaces. In the third talk, I will explain the Frobenius structure conjecture by Gross-Hacking-Keel concerning the enumeration of rational curves in log Calabi-Yau varieties, and describe how we can use the enumeration of non-archimedean holomorphic disks to prove the conjecture in dimension 2, as well as in certain higher dimensional cases. It is joint work with Sean Keel.

- Talk 1: Non-archimedean SYZ fibration
- Talk 2: Enumeration of open curves in log Calabi-Yau surfaces
- Talk 3: The Frobenius structure conjecture

Research talks:

François Charles: *Cohomology of arithmetic varieties and arithmetic ampleness*

Theta-invariants of infinite-dimensional lattices were introduced by Bost. I will describe how they occur naturally in the cohomology of arithmetic schemes, and will use them to reprove basic results on arithmetic ampleness. This is joint work with Bost.

Frédéric Déglise: *Mixed motives and t-structures*

The theory of mixed motives has made tremendous progress in the last twenty years, under the impulse of Beilinson and Voevodsky. The talk will start with a short summary of these advances.

For the time being, only triangulated mixed motives are available: the existence of the "motivic t-structure", from which one would deduce abelian mixed motives, is the main conjecture of the field. Though this t-structure is conjectural, one has another t-structure on triangulated mixed motives over a perfect field, Voevodsky's homotopy t-structure. I will explain how this t-structure allows one to express the main conditions of existence of the motivic t-structure, extending Beilinson and Soulé's vanishing conjecture.

This motivates an extensive study of the homotopy t-structure. I will end the talk with the results in that direction that I obtained in collaboration with Bondarko. The heart of the homotopy t-structure, relative to an arbitrary base, shares several features of the category of perverse sheaves.

Philippe Eyssidieux: *How to construct interesting Kähler Groups?*

The Corlette-Simpson non abelian Hodge theory gives many restrictions on the representation theory of Kähler groups but actually fails to provide construction methods. We shall explain how orbifold compactifications of quasi-Kähler manifolds with an interesting fundamental group provide potentially interesting orbifold Kähler groups, what tools can be used to handle them, and give some non trivial examples of orbifold Kähler groups . This raises the difficult question whether the class of orbifold Kähler and Kähler groups coincide - they do in the residually finite case.

Javier Fresán: *A geometric construction of the polylogarithm motive*

(Joint work with Clément Dupont) Inspired by the integral representation of the polylogarithm function, we give a geometric construction (i.e.as a relative cohomology) of the polylogarithm motive in the category of mixed Tate motives over the projective line minus three points. We then study its realisations and the limiting motives at the punctures.

Jingjun Han: *Birational boundedness of rationally connected Calabi–Yau 3-folds*

We will show that rationally connected klt Calabi–Yau 3-folds form a birationally bounded family, and such 3-folds with mld bounding away from 1 are bounded modulo flops. This is a joint work with Weichung Chen, Gabriele Di Cerbo, Chen Jiang and Roberto Svaldi.

Andreas Höring: *Algebraic integrability of foliations with numerically trivial bundle*

The Beauville-Bogomolov decomposition of smooth projective manifolds is a cornerstone of the classification of higher-dimensional varieties. For varieties with mild singularities (e.g. minimal models) the analytic tools used in the proof are not available, so one has to look for a different strategy. In this talk I will explain how the algebraic integrability of foliations enters the picture and how a positivity result for reflexive sheaves allows to extend the decomposition theorem to the singular setting. This is joint work with Thomas Peternell.

Zhi Jiang: *Cohomological rank functions and Severi type inequalities*

I will discuss some properties of cohomological rank functions on abelian varieties introduced by Barja-Pardini-Stoppino and Pareschi-Jiang and discuss how to use these functions to get Severi type inequalities.

Chi Li: *On the stability of extensions of tangent sheaves on Kähler-Einstein Fano/Calabi-Yau pairs*

I will talk about generalizations of a stability result of Tian for K-semistable Fano manifolds, and applications to local Euler numbers of log canonical surface singularities and Miyaoka-Yau type inequalities.

Zhiyuan Li: *Unirationality of supersingular irreducible symplectic varieties*

The supersingular K3 surfaces are first introduced by Artin via the formal Brauer groups. Artin has conjectured that all supersingular K3 surface are unirational. Recently, this conjecture has been confirmed by Liedtke when $p > 3$. In this talk, we introduce several notions of supersingular irreducible symplectic (IS) varieties over fields of positive characteristic p, which are conjecturally equivalent. By analogy with K3 surfaces, there is a natural generalization of Artins conjecture for supersingular IS varieties. We will show that the conjectures hold for most known examples of IS varieties. This is a joint work with Lie Fu.

Ruochuan Liu: *p-adic Riemann-Hilbert correspondence, de Rham comparison and periods on Shimura varieties*

In the previous work with Xinwen Zhu we construct a p-adic analogue of the classical Riemann-Hilbert correspondence. As a by-product the de Rham periods of a general Shimura variety are obtained. In a recent joint work with Hansheng Diao, Kai-Wen Lan and Xinwen Zhu, we further establish a logarithmic version of the correspondence which enables us to establish the de Rham comparison theorem with coefficients for quasi-projective varieties and compare the de Rham periods and complex periods for a general Shimura variety.

Olivier Wittenberg: *On the Lüroth problem for real varieties*

(Joint work with Olivier Benoist.) The Lüroth problem asks whether unirational varieties are rational. It has a positive answer for complex curves and surfaces; negative answers for complex threefolds have been understood since the 70's. I will discuss the Lüroth problem for real algebraic varieties that are geometrically rational and explain a counterexample not accounted for by the topology of the real locus or by a nontrivial unramified cohomology group over the reals.

Junyi Xie: *The geometric Bogomolov conjecture*

With Cantat, Habegger and Gao, we prove the geometric Bogomolov conjecture over a function field of characteristic zero. This generazed the recent work of Habegger and Gao, who proved the geometric Bogomolov conjecture over a function field of a curve.

Kang Zuo: *Arithmetic Simpson Correspondence and GL_2-Motivic Local Systems over $\mathbb{P}^1\backslash\{0, 1, \lambda, \infty\}$*

Given an arithmetic log scheme $(\mathcal{X}, \mathcal{D})$ over a number ring $\mathcal{O}_K$, we propose an arithmetic Simpson correspondence between the category of stable arithmetic periodic Higgs bundles and the category of geometric irreducible motivic local systems over $(\mathcal{X}, \mathcal{D})$.

As an application, we investigate the geometric irreducible GL_2-motivic local systems over $\mathbb{P}^1$ with prescribed singularities on a given set $\mathcal{D} := \{0, 1, \lambda, \infty\}$. Defining the associated elliptic curve $\mathcal{C}_\lambda$ as the double cover of $\mathbb{P}^1$ ramified on $\mathcal{D}$. The proposed arithmetic Simpson correspondence predicts that there exists a bijective correspondence between the set of PGL_2-motivic local systems over $(\mathbb{P}^1, \mathcal{D})$ and the set of torsion points on $\mathcal{C}_\lambda$ defined by taking those Higgs bundles (E, θ) whose Higgs fields θ vanish exactly on torsion points of $\mathcal{C}_\lambda$. For example, there exist 26 elliptic curves over

$(\mathbb{P}^1, \mathcal{D})$ with zeros $(\theta)_0$ of Kodaira-Spencer maps as torsion points of order 1, 2, 3, 4 or 6; there exist 156 abelian surfaces with a real multiplication L of degree 2 over $(\mathbb{P}^1, \mathcal{D})$ with $(\theta)_0$ of order 5, 8, 10, 12 or 24; ... so on.

On the other hand, we do find 26 elliptic curves over $(\mathbb{P}^1, \mathcal{D})$ whose zeros of Kodaira-Spencer maps are exactly the torsion points of $\mathcal{C}_\lambda$ of the expected orders.

Kontsevich has observed a relation between the set of isomorphic classes of $\mathrm{GL}_2(\bar{\mathbb{Q}}_l)$-local systems over $(\mathbb{P}^1, D)$ over $\mathbb{F}_q$ and the set of rational points on C_λ over $\mathbb{F}_q$ via the work of Drinfeld on the Langlands program over function field. It looks quite mysterious. There should exist a relation between periodic Higgs bundles in the p-adic world and the Hecke-eigenforms in the ℓ-adic world via Abe's solution of Deligne conjecture on ℓ-to-p companions. This is an on-going program joint with J. Lu, X. Lu, R.R. Sun and J.B. Yang.